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## **АКТУАЛЬНІ ПРОБЛЕМИ ФУНДАМЕНТАЛЬНИХ НАУК**

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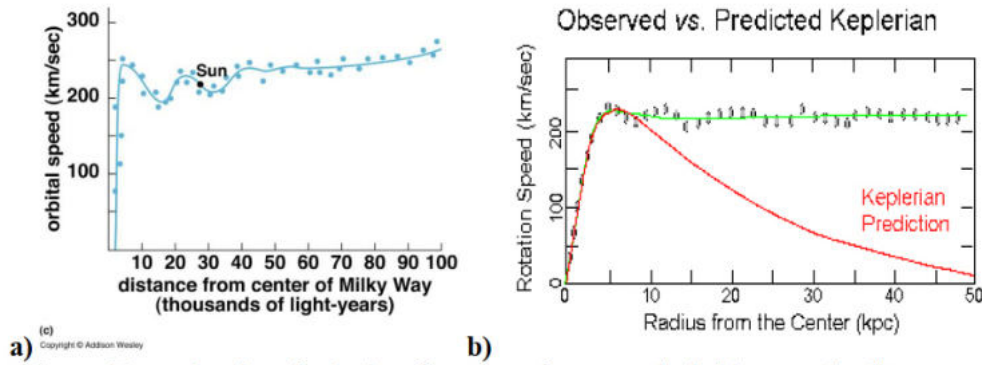
## SOLUTION OF EQUATIONS OF THE GALAXY GRAVITATIONAL FIELD

**Danylchenko, Pavlo<sup>2</sup>**

The general solution of the equations of the gravitational field of the galaxy with an additional variable parameter  $n$  is found. The additional variable parameter  $n$  determines in GR the distribution of the average mass density mainly in the friable galactic nucleus. The velocity of the orbital motion of stars is close to Kepler only for  $n > 2^{25}$ . At  $n < 2^{15}$ , it is slightly less than the highest possible velocity even at the edge of the galaxy. The maximum allowable value of the average mass density of a substance outside the friable galactic nucleus negligibly weakly depends on the parameter  $n$  in GR. And it can be arbitrarily small. Therefore, in relativistic gravithermodynamics, in contrast to GR, there can be no shortage of baryonic mass.

**Key words:** General Relativity, relativistic gravithermodynamics, Keplerian velocities, non-baryonic dark matter, intranuclear temperature and pressure, friable galactic nucleus.

Laws of motion of single astronomical objects, found by Kepler, are based on gravitational influence of mainly central massive body. According to those laws, the velocity of rotation of galactic objects should decrease in inverse ratio to the square root of the distance to galaxy center. However, observations reveal the different picture: this velocity remains quasi constant on quite far distance from galaxy center for many galaxies, including ours [Bennett et al., 2012]:



**Figure.** Dependencies of velocity of rotation of astronomical objects on the distance to gravity center: (a) *our Milky Way galaxy* [Bennett et al., 2012], (b) *comparing to prognosed Keplerian velocities* [Thompson, 2011]).

The General Relativity (GR) gravitational field equations de facto correspond to spatially inhomogeneous thermodynamic states of only utterly cooled down matter. The similar to them equations of relativistic gravithermodynamics (RGTD) correspond to spatially inhomogeneous thermodynamic states of gradually cooling down matter. That is why in the RGTD the four-momentum is formed not by enthalpy but by the ordinary internal energy of matter (multiplicative component of its total energy). According to this, in the tensor of energy-momentum of the RGTD not only intranuclear pressure  $p_N$  but also intranuclear temperature  $T_N$  is taken into account:

$$b'/abr - r^{-2}(1-1/a) + \Lambda = \kappa(T_N S_N - p_N V_N)/V = \kappa \mu_{cr} c^2 [1/\sqrt{b} - \sqrt{b}] = \kappa \mu_{cr} c^2 (1/b - 1), \quad (1)$$

$$a'/a^2 r + r^{-2}(1-1/a) - \Lambda = \kappa[\mu_{cr} c^2 \sqrt{b} + (T_N S_N - p_N V_N) v_g^2 / V b c^2] / (1 - v_g^2 / b c^2) = \kappa \mu_{cr} c^2 [1 + v_z^2 / b(c^2 - v_z^2)]. \quad (2)$$

The defined by the same spatial distribution average relativistic density of corrected relativistic mass of galaxy matter in RGTD has the following form:  $\mu_r = \mu_{cr} \sqrt{b} [1 + v_z^2 / b(c^2 - v_z^2)]$ , where:

<sup>2</sup> © Danylchenko, Pavlo, 2021, E-mail: pavlo@vingeo.com



$$\sqrt{b} = v_z / c = a^{-1/2} \left[ 1 + (\kappa c^2 / 2) \int_{r_e}^r (m_{cr} / V) (1 - v_z^2 c^{-2})^{-1} a^{3/2} r dr \right],$$

$v_g = v_z \sqrt{b}$  and  $v_z$  are observable and zonal velocity of rotation of astronomical objects by the clock of the outer space that surrounds them and is not dragged by the motion of astronomical objects themselves,  $\mu_{cr} = m_{cr} / V$ ,  $V$  is volume of matter,  $m_{cr} = b^{-1/2} m$  is intrinsic value of the mass of matter that corresponds to “critical” equilibrium value of the ordinary internal energy of matter ( $b=1$ ), and  $v_{lg}$  is maximum possible (extreme) value of velocity of matter in the outer space of the galaxy [Danylenko, 2009; 2020: 5].

According to this we find the square of the rotation velocity of astronomical object relatively to the galaxy center according to the equations of gravitational field of RGTG:

$$\begin{aligned} [v_z^2]_{RGTG} &= c^2 r \frac{d \ln(v_{lg} / c)}{dr} = \frac{c^2 a}{2} \left\{ \eta + \left[ \frac{\kappa (T_N S_N - p_N V_N)}{V} - \frac{2\Lambda}{3} \right] r^2 \right\} = \frac{c^2 \{ \eta + [\kappa \mu_{cr} c^2 (1/\sqrt{b} - \sqrt{b}) - 2\Lambda/3] r^2 \}}{2(1-\eta-\Lambda r^2/3)} = \\ &= \frac{c^2}{2(1-\eta-\Lambda r^2/3)} \left[ \frac{\kappa \mu_r c^2 r^2}{b + (1-b)v_z^2 c^{-2}} - \chi_0 r - \frac{2\Lambda r^2}{3} \right] = \frac{c^2}{2(1-\eta-\Lambda r^2/3)} \left[ \frac{\eta + \chi r}{b + (1-b)v_z^2 c^{-2}} - \frac{2\Lambda r^2}{3} \right] > [v_z^2]_{GR}, \quad (3) \end{aligned}$$

where:  $\chi = (1-b)(1-v_g^2/bc^2) \chi_0 = \kappa \mu_0 c^2 (1-b)(1-v_z^2 c^{-2}) [r_e \exp(-r/r_e) + 2\pi \sigma r_m \cos(2\pi r/r_m)]$ .

As we can see, at the same radial distribution of the average density of the mass  $\mu_r$  of baryonic matter the circular velocities of rotation of astronomical objects relatively to the galaxy center are much bigger in RGTG than in GR. And this is, of course, related to the fact that:

$$(T_N S_N - p_N V_N) / V = \mu_{cr} c^2 (1/b - 1) = \mu_{cr} c^2 (1/\sqrt{b} - \sqrt{b}) > p.$$

Therefore, we can get rid of the imaginary necessity of dark non-baryonic matter in galaxies that follows from the equations of GR gravitational field if we analyze the motion of their astronomical objects using the equations of gravitational field of RGTG.

If we do not take into account local peculiarities of distribution of average density of the mass in galaxies and examine only the general tendency of typical dependence of the orbital velocity of their objects on radial distance to the galaxy center, then the following dependence of this velocity on parameter  $b$  and, thus on radial distance  $r$ , can be matched with the graphs:

$$v_z = v_{z\max} \{ [(b/b_e)^n + (b_e/b)^n] / 2 \}^{-1/2} = c \{ [2n \ln(r/r_e)]^2 + (c/v_{z\max})^4 \}^{-1/4}, \quad (4)$$

where:

$$\begin{aligned} b &= b_e \left[ (v_{z\max} / v_z)^2 \pm \sqrt{(v_{z\max} / v_z)^4 - 1} \right]^{1/n} = b_e \left[ \pm 2n v_{z\max}^2 c^{-2} \ln(r/r_e) + \sqrt{1 + [2n v_{z\max}^2 c^{-2} \ln(r/r_e)]^2} \right]^{1/n}, \quad (5) \\ r &= r_e \exp \left[ \pm (c^2 / 2n) \sqrt{v_z^{-4} - v_{z\max}^{-4}} \right] = r_e \exp \left\{ \pm (c^2 v_{z\max}^{-2} / 4n) [(b/b_e)^n - (b_e/b)^n] \right\}, \end{aligned}$$

and:  $r_e$  is radius of the conventional friable galactic nucleus, on the surface of which the orbital velocity of objects can take its maximum possible value  $v_{ze}(b_e) = v_{z\max}$ .

The smaller value  $b$  corresponds to the larger value  $n$  of the index of density of friable galactic nucleus on the same big radial distances. However, only when values are extremely large  $n > 2^{25}$  the significantly lesser average density of matter beyond the friable galactic nucleus takes place and that is why the dependence of orbital velocities of galactic objects on radial distances can be close to Keplerian. When the parameter values are  $n < 2^{15}$  the orbital velocities of extra-nuclear objects are, according to (4), quite close to their maximum values  $v_{z\max} < 225 \text{ km/s}$  (Fig. b)) on quite big radial distances  $r/r_e < 20$ :

$$\Delta v_z = v_{z\max} - c \{ [2n \ln(r/r_e)]^2 + (c/v_{z\max})^4 \}^{-1/4} < 0,683 \text{ [km/s]}.$$

This, of course, is related to the fact that big gradients of gravitational field on the periphery of such galaxies are formed not by their nuclei but by all large set of their objects.

Then, taking into account the negligible smallness of cosmological constant and of the pressure in the outer space of the galaxy, the following typical radial distribution of average density of mass of matter in the galaxy will take place in GR:



$$[\mu]_{GR} \approx \frac{1}{\kappa^2} \left[ \frac{a'}{a^2 r} + \frac{1}{r^2} \left( 1 - \frac{1}{a} \right) \right] \left( 1 - \frac{v_g^2}{bc^2} \right) = \frac{2v_z^2(1-v_z^2c^{-2})[1+2v_z^2c^{-2}-4n^2v_z^4c^{-4}\ln(r/r_e)]}{\kappa^4 r^2(1+2v_z^2c^{-2})^2} \approx \frac{v_z^2}{4\pi G r^2},$$

where  $G = \kappa c^4/8\pi$  is Newton's gravitational constant, and according to (4):

$$a \approx 1 + 2v_z^2c^{-2} = 1 + 2\{[2n\ln(r/r_e)]^2 + (c/v_{z\max})^4\}^{-1/2}. \quad (6)$$

Thus, according to GR, the bigger the index  $n$  and the lesser the value of parameter  $b_e$ , the lesser is maximum possible value of average density of mass of the matter on the edge of the galaxy. However, when  $v_{z\max} = 225 \text{ km/s}$ ,  $r_e = 5 \text{ kpc}$ ,  $r_{\lim}/r_e = 20$ ,  $n = 2^{15}$  ( $v_{z\lim} = 224,317294 \text{ km/s}$ ) and  $b_e = 0,99999551225433188$  ( $b_{\lim} = 0,999999888026921702$ ):  $[\mu_{\lim}]_{GR} = 6,276 \cdot 10^{-24} \text{ kg/m}^3$  is only 0,4% smaller than its approximate value. And, therefore, due to  $v \ll c$  it quite weakly depends on the index  $n$  of the density of friable galactic nucleus.

In RGTD (taking into account the negligible smallness of only cosmological constant) the completely different typical radial distribution of the average density of mass of the matter in the galaxy takes place:

$$[\mu]_{RGTD} \approx \frac{b\delta}{\kappa^2 r^2 a(1-b)} = \frac{b[2v_z^2c^{-2} - (a-1)]}{\kappa^2 r^2 a(1-b)} = \frac{b[4v_{z\max}^2c^{-2}b_e^n b^n - (a-1)(b^{2n} + b_e^{2n})]}{\kappa^2 r^2 a(1-b)(b^{2n} + b_e^{2n})}, \quad (7)$$

according to which in the case of fulfillment of condition (10) it becomes infinitely small. The tendency to 1 of not only parameter  $a$ , but also parameter  $b$ , prevents the limitless decrease to zero of average density of mass of matter on the edge of the galaxy. That is why in RGTD, in contrast to GR, there cannot be in principle any shortage of baryonic mass not only in the center, but also on the edge of the galaxy.

Taking into account that in the outer space on the periphery of the galaxy  $a_{\lim} - 1 \approx 1 - b_{\lim}$  and, thus,  $a_{\lim} = 1,00000111973203677$  (when  $2v_{z\lim}^2c^{-2} = 1,11973203777 \cdot 10^{-6}$ ), having the same initial data we can find the acceptable value of the average density of mass of matter on the edge of the galaxy:  $[\mu_{\lim}]_{RGTD} = 5 \cdot 10^{-26} \text{ kg/m}^3$ . However, of course, when we have value  $b_e$ , that guarantees  $\delta_{\lim} < 10^{-15}$ , the significantly smaller average density of mass of the matter on the edge of the galaxy can take place in RGTD. When  $n = 1$  ( $v_{z\lim} = 224,99999999936 \text{ km/s}$ ) and the same value  $\delta_{\lim} = 10^{-15}$  ( $b_e = 0,99999606363264543$ ,  $b_{\lim} = 0,999999436721227408$ )  $[\mu_{\lim}]_{RGTD} = 1,4 \cdot 10^{-27} \text{ kg/m}^3$ .

## Conclusion

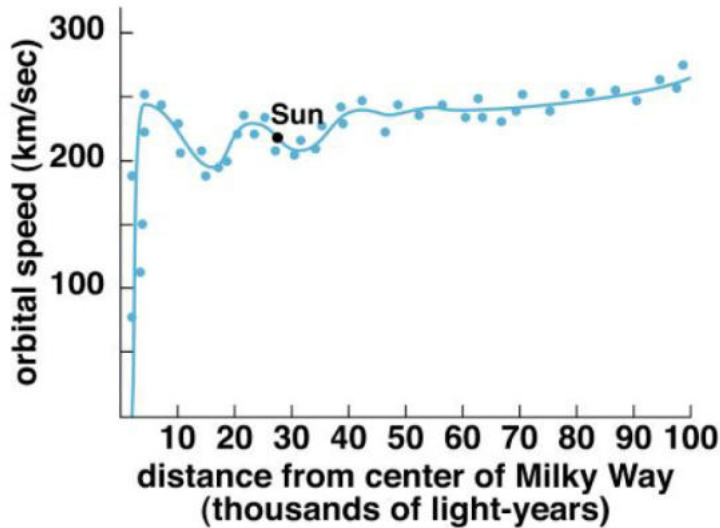
The general solution of the equations of the gravitational field of the galaxy with an additional variable parameter  $n$  is found. The additional variable parameter  $n$  determines in GR the distribution of the average mass density mainly in the friable galactic nucleus. The velocity of the orbital motion of stars is close to Kepler only for  $n > 2^{25}$ . At  $n < 2^{15}$ , it is slightly less than the highest possible velocity even at the edge of the galaxy. If the energy-momentum tensor is formed not on the basis of external thermodynamic parameters, but on the basis of intranuclear gravithermodynamic parameters of the substance, then the dependence of the average mass of the substance on the value of the parameter  $n$  becomes very significant. The permissible value of the average mass density of matter outside the friable galactic nucleus is determined by the value of the parameter, which is responsible for the curvature of space. And it can be arbitrarily small. Therefore, in relativistic gravithermodynamics, in contrast to GR, there can be no shortage of baryonic mass in principle. And, therefore, the presence of non-baryonic dark matter in the Universe is not necessary.

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### Observed vs. Predicted Keplerian

