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THE SOLUTIONS OF EQUATIONS OF GRAVITATIONAL FIELD FOR QUANTUM QUASI-EQUILIBRIUM COOLING DOWN GASES

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The quantum equation of gravitational field have been found, the solutions of which set the spatial distribution of gravitational radius of matter in its every new gravithermodynamic (GTD) state with the polynomial function with the next more high degree. The indicator of the degree of this function of continuously cooling down matter can successively take only integer and semi-integer values. That is why the process of cooling down of the whole GTD-bonded matter is the quantum process that is caused by its spontaneous transition to the polynomial function with more high value of degree and, therefore, to the next quantum state.

Keywords: gravithermodynamics, thermodynamics, GR, collective space-time microstate, Gibbs microstate, entropy, hidden variable, quantum cooling down gases.

Due to the fact that the whole gravithermodynamically bonded matter forms the collective spatial-temporal microstates (Gibbs microstates) the spatial integration of equations of gravitational field has the physical sense only for the specific moment of intrinsic time of matter and only in the (inseparable from it) intrinsic space. Exactly the cardinal absence of the velocity of motion of matter in integrated equations makes the problem of relativistic invariance of thermodynamical parameters and potentials of matter non-actual. Since in quasiequilibrium cooling down clusters of homogenous gas the functions of time $t(R_T)$ and of rigidly related to cooling down gas radial coordinate $r(A_p)$ ($v_r=dr/dt=0$) perform the time-like gas parameter $R_T=pV/T=\mathbf{const}(r)$ and the indicator of hierarchic complexity of gas $A_p=ST/R_T=ST^2/pV=\mathbf{const}(t)$ correspondingly, the spatial integration of equations of its gravithermodynamic state should be performed for the same value of parameter while the temporal integration should be performed for the same value of indicator of hierarchic complexity of any concrete microvolume of the gas.

Given this, the gas cluster that is cooling down in quasi-equilibrium can be matched in general relativity (GR) to thermodynamic frame of references (FR) that corresponds to Schwarzschild parameters of equations of gravitational field. Since for quasi-equilibrium cooling down gases ($(T/V)dS/dr+dp/dr+(\mu_{gr0}c^2+p)b'/2b=0$ ($R_T=\mathbf{const}(r)$), then for them not only in GR, but also and in relativistic gravithermodynamics (RGTD) [1] we will have:

$$(dS/dr)/R_T+(dp/dr)/p=-b'\tilde{\beta}_H/2\tilde{\beta}_{pV}b=-[1+(\tilde{k}\tilde{l}\tilde{m}-1)/\tilde{k}(\tilde{l}-1)]v'_{cv}/v_{cv}=\{[\tilde{l}(\tilde{m}+1)-(\tilde{k}+1)/\tilde{k}]/(\tilde{l}-1)T\}(dT/dr).$$

From here: $(S-S_w)/R_T+\ln(p/p_w)=-(\tilde{\beta}_H/2\tilde{\beta}_{pV})\ln(b/b_w)=\{[\tilde{k}\tilde{l}(\tilde{m}+1)-(\tilde{k}+1)]/\tilde{k}(\tilde{l}-1)\}\ln(T/T_w)$,

$$S/R_T=(\tilde{\beta}_H/\tilde{\beta}_{pV})\ln(R_T T/U_{cr})-\ln(p/p_l)-\ln\tilde{\beta}_{pV}/\tilde{\beta}_{pV},$$

where: $\tilde{\beta}_{pV}(R_T)\tilde{n}(R_T)=(p_{00}/p_{cr})\exp(S_{00}/R_T)$; $\tilde{\beta}_{pV}=pV/U_0=p/\mu_{gr0}=\tilde{k}(\tilde{l}-1)/(\tilde{k}\tilde{l}\tilde{m}-1)=\mathbf{const}(r)$ (according to Boyle-Mariotte law); $\tilde{\beta}_{ST}=ST/(\hat{S})U_0=(\tilde{k}\tilde{m}-1)/(\tilde{k}\tilde{l}\tilde{m}-1)=\mathbf{const}(r)$; $\tilde{\beta}_H=H_0/U_0=1+\tilde{\beta}_{pV}$, $\tilde{\mu}_l=p_{cr}c^{-2}\tilde{n}/\tilde{\beta}_{pV}$ is the limit value of density of the gravitational mass of gas; $\tilde{k}$, $\tilde{l}$, $\tilde{m}$, $\tilde{n}$ are mathematical expectations of thermodynamic hidden variables [1].

Obviously, the clusters of quasiequilibrium cooling down gas ($U_{ad}=0$) have also the spatial homogeneity of gravithermodynamic intrinsic value (eigenvalue) of Gibbs free energy G^*_{00} (similarly to the spatial homogeneity of thermodynamic intrinsic value of enthalpy H^*_{00} of cooled down to the limit matter, as it was shown by Tolman [2]):

$$G^*_{00}=(v_{lc}/c)G=(v_{lc}/c)[U_0+U_{ad}+pV-TS]=U_{cr}\psi_{m0}[1+\tilde{\beta}_{pV}-\tilde{\beta}_{TS}\ln(f_I^{\tilde{l}}N_I^{1-\tilde{l}})]=U_{00}[1+\tilde{\beta}_{pV}(1-S/R_T)],$$

where: $R_T=\mathbf{const}(r)$, , $v_{lc}=\sqrt{b}c=v_l/\Gamma_m=c\psi_{m0}/f_I$, $N_I=f_I^{-\tilde{l}/(1-\tilde{l})}\exp[S\tilde{\beta}_{pV}/\tilde{\beta}_{ST}R_T(1-\tilde{l})]$. Therefore:

$$V=(U_{cr}/\tilde{p}_l)f_I^{-1/\tilde{k}}N_I^{(1-\tilde{k}\tilde{m})/\tilde{k}}=(U_{cr}/\tilde{n}p_{cr})(b\psi_{m0}^{-2})^{1/2\tilde{\beta}_{pV}}\exp(S/R_T)=(V_{00}/\tilde{n})\exp(S/R_T)b^{1/2\tilde{\beta}_{pV}},$$

$$p=\widetilde{p}_l\widetilde{\beta}_{pV}f_I^{(1+\widetilde{k})/\widetilde{k}}N_I^{(\widetilde{k}\widetilde{m}-1)/\widetilde{k}}=p_{cr}\widetilde{n}\widetilde{\beta}_{pV}(b\psi_{m0}^{-2})^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}\exp(-S/R_T)=p_{00}\widetilde{n}\widetilde{\beta}_{pV}\exp(-S/R_T)b^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}},$$

$$T=\widetilde{\beta}_{pV}U_{cr}f_I/R_T=(\psi_{m0}U_{cr}\widetilde{\beta}_{pV}/R_T)b^{-1/2}=T_{00}b^{-1/2},\ U_0=\psi_{m0}U_{cr}b^{-1/2}=(T_{00}R_T/\widetilde{\beta}_{pV})b^{-1/2}=U_{00}b^{-1/2},$$

$$\mu_{gr0}c^2=(\widetilde{n}T_{00}R_T/V_{00}\widetilde{\beta}_{pV})\exp(-S/R_T)b^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}=\widetilde{\mu}_c c^2 b^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}=r_g'r^{-2}/\kappa,$$

$$G_0/V=(\widetilde{n}T_{00}/V_{00})(R_T\widetilde{\beta}_H/\widetilde{\beta}_{pV}-S)\exp(-S/R_T)b^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}=(\widetilde{\mu}_c c^2+\widetilde{\sigma}_c)b^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}=(1+\widetilde{\beta}_{pT})r_g'r^{-2}/\kappa,$$

where: $G_{00}=G_0b^{1/2}=T_{00}(R_T\widetilde{\beta}_H/\widetilde{\beta}_{pV}-S)=U_{00}(1+\widetilde{\beta}_{pT})$, $U_{00}=U_0b^{1/2}$; $\widetilde{\beta}_{pT}=(pV-TS)/U_0=\widetilde{\beta}_{pV}(1-S/R_T)$, $T_{00}=\psi_{m0}U_{cr}\widetilde{\beta}_{pV}/R_T=Tb^{1/2}$, $V_{00}=(U_{cr}/p_{cr})\psi_{m0}^{-1/\widetilde{\beta}_{pV}}$, $p_{00}=p_{cr}\psi_{m0}^{\widetilde{\beta}_H/\widetilde{\beta}_{pV}}$; $\widetilde{\mu}_c=\widetilde{\mu}_l b_l^{\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}=$ $=(\widetilde{n}T_{00}R_T/c^2V_{00}\widetilde{\beta}_{pV})\exp(-S/R_T)$ and $\widetilde{\sigma}_c=(\widetilde{n}T_{00}/V_{00})(R_T-S)\exp(-S/R_T)=\widetilde{\mu}_c c^2\widetilde{\beta}_{pV}(1-S/R_T)=\widetilde{\mu}_c c^2\widetilde{\beta}_{pT}$ are mathematical expectations (when $b=1$) of mass density and the difference of densities of Gibbs free energy and gas mass correspondingly; U_0 and G_0 are multiplicative components of internal energy and Gibbs free energy correspondingly; ψ_{m0} is the parameter that connects internuclear and intranuclear characteristics of the gas; $b=v_{lc}^2c^{-2}$.

Due to the relativistic invariance of thermodynamic parameters and characteristics of matter the equations of gravitational field for quasiequilibrium cooling down spherically symmetric gas cluster and in commoving with it FR can be expressed in GR in the following way:

$$b'/abr-r^{-2}(1-1/a)+\Lambda(R_T)=\kappa(p-ST/V)=\kappa(\widetilde{n}T_{00}/V_{00})(R_T-S)\exp(-S/R_T)b^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}=\kappa\widetilde{\sigma}_c b^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}},$$

$$a'/a^2r+r^{-2}(1-1/a)-\Lambda(R_T)=\kappa\mu_{gr0}c^2=\kappa(\widetilde{n}T_{00}R_T/V_{00}\widetilde{\beta}_{pV})\exp(-S/R_T)b^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}=\kappa\widetilde{\mu}_c c^2 b^{-\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}=r^{-2}r_g',$$

$$\frac{1}{a(r)}=\left(\frac{\partial r}{\partial\widetilde{r}}\right)^2=\chi(r)b(r)=-\kappa c^2\widetilde{\mu}_c b(1+\widetilde{\beta}_{pT})\int_{r_0}^{r}b^{-(1+3\widetilde{\beta}_{pV})/2\widetilde{\beta}_{pV}}rdr=\frac{1}{a_b}-\frac{\Lambda}{3}r^2=1-\left(1-\frac{\Lambda}{3}r_0^2\right)\frac{r_0}{r}-\frac{\kappa c^2\widetilde{\mu}_c}{r}\int_{r_0}^{r}b^{-(1+\widetilde{\beta}_{pV})/2\widetilde{\beta}_{pV}}r^2dr-\frac{\Lambda}{3}r^2,$$

$$1/a(r)=1-r_g(r)/r-r^2\Lambda/3=1/a_b(r)-r^2\Lambda/3,\qquad b(r)=1-r_g(r)/r-r^2\Lambda/3=1/a_b(r)-r^2\Lambda/3,$$

where: $r_g(r)=r(1-1/a_b)=(1-r_0^2\Lambda/3)r_0+\kappa\widetilde{\mu}_l c^2 b_l^{(1+\widetilde{\beta}_{pV})/2\widetilde{\beta}_{pV}}\int_{r_0}^{r}b^{-(1+\widetilde{\beta}_{pV})/2\widetilde{\beta}_{pV}}r^2dr$ $(1/a_0=0)$ is the value of gravitational radius of matter that is covered by the sphere with radius $r\geq r_0$; r_0 is minimum possible value of Schwarzschild radial parameter; $b=\widetilde{\beta}_{pV}^2\psi_{m0}^2U_{cr}^2R_T^{-2}T^{-2}=(r_g'r^{-2}c^2/\kappa\widetilde{\mu}_c)^{-2\widetilde{\beta}_{pV}/\widetilde{\beta}_H}$.

According to this we will receive the differential equation of the second order for the gravitational radius of gas cluster that is cooling down in quasi-equilibrium:

$$-2(\widetilde{\beta}_{pV}/\widetilde{\beta}_H)(r_g''r/r_g'-2)(1-r_g/r-r^2\Lambda/3)-r_g/r+2r^2\Lambda/3=\widetilde{\beta}_{pT}r_g',$$

$$r_g''/r+[r_g'^2\widetilde{\beta}_{pT}\widetilde{\beta}_H/2\widetilde{\beta}_{pV}r-2r_g'r^{-2}+(r_g-2r^3\Lambda/3)r_g'\widetilde{\beta}_H/2\widetilde{\beta}_{pV}]/(r-r_g-r^3\Lambda/3)=0.$$

When there are new parameters this equation is transformed into differential equation of the first order:

$$u'-ru^2/n\varphi+\psi\varphi'/\varphi=u'-(1-\widetilde{\beta}_H\widetilde{\beta}_{pT}/2\widetilde{\beta}_{pV})ru^2/\varphi+r_g'r^{-2}(r_g-2\Lambda r^3/3)(r-r_g-\Lambda r^3/3)^{-2}(1+3\widetilde{\beta}_{pV})/2\widetilde{\beta}_{pV}=0,$$

where: $u=\psi\varphi=r_g'\varphi/r(r-r_g-\Lambda r^3/3)$, $\psi=r_g'/r(r-r_g-\Lambda r^3/3)$,

$$n=2\widetilde{\beta}_{pv}/(2\widetilde{\beta}_{pV}-\widetilde{\beta}_H\widetilde{\beta}_{pT})=2/[(1+S/R_T)-\widetilde{\beta}_{pV}(1-S/R_T)]=2pVU_0/[(pV+ST)U_0-(pV-ST)pV],$$

$$u'=\psi'\varphi+\psi\varphi'=\frac{r_g''\varphi}{r(r-r_g-\Lambda r^3/3)}-\frac{r_g'\varphi(1-r_g'-\Lambda r^2)}{r(r-r_g-\Lambda r^3/3)^2}-\frac{r_g'\varphi}{r^2(r-r_g-\Lambda r^3/3)}+\frac{r_g'\varphi'}{r(r-r_g-\Lambda r^3/3)}=\frac{r\psi^2\varphi}{n},$$

$$\psi'=r\psi^2/n-\psi\varphi'/\varphi=r\psi^2/n-r_g'(1+3\widetilde{\beta}_{pV})(r_g-2r^3\Lambda/3)(r-r_g-r^3\Lambda/3)^{-2}r^{-2}/2\widetilde{\beta}_{pV}.$$

Since in the central zone of gas cluster ($\varphi\approx 1$, $\psi'\approx r\psi^2/n$), we will determine the approximate solution of differential equation $\widetilde{\psi}'=r\widetilde{\psi}^2/n$ exactly for this zone:

$$2n\left(\frac{1}{\widetilde{\psi}_0}-\frac{1}{\widetilde{\psi}}\right)=2n\left[\frac{r_0^2(1-r_{g0}/r_0-\Lambda r_0^2/3)}{r'_{g0}}-\frac{r^2(1-r_g/r-\Lambda r^2/3)}{r'_g}\right]=\frac{2n}{\kappa\widetilde{\mu}_c c^2}\left(\frac{1}{a_0 b_0^{\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}}-\frac{1}{ab^{\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}}\right)=r^2-r_0^2 .$$

From here: $r'_g=2n(rr_g-r^2+r^4\Lambda/3)/(r^2-\rho_0^2)$, $r_g=(r^2-\rho_0^2)^n/z$, where: $\rho_0^2=r_0^2-2n(r_0/r'_{g0})(r_0-r_{g0}-\Lambda r_0^3/3)$, and the parameter z is determined from: $-d(1/z)/dr=z'z^{-2}=2n(r^2-r^4\Lambda/3)(r^2-\rho_0^2)^{-(n+1)}$.

The obtained differential equation is the quantum equation of gravitational (gravithermodynamic) field since its solutions are the polynomial functions the degree indicators of which ($n=3/2,\ 2,\ 5/2,\ 3\ ...\ \infty$) can take only integer or semi-integer values:

$$r_g=(r^2-\rho_0^2)^n\left[\frac{1}{z_0}-2n\int_{r_0}^{r}\frac{r^2-r^4\Lambda/3}{(r^2-\rho_0^2)^{n+1}}dr\right]=(r^2-\rho_0^2)^n\left\{\frac{1}{z_0}-2n\int_{r_0}^{r}\left[-\frac{\Lambda/3}{(r^2-\rho_0^2)^{n-1}}+\frac{1}{(r^2-\rho_0^2)^{n}}-\rho_0^2\frac{1+\rho_0^2\Lambda/3}{(r^2-\rho_0^2)^{n+1}}\right]dr\right\}=$$

$$=\left\{\frac{(r^2-\rho_0^2)^n}{z_0}+\frac{1}{\rho_0^2}\left[(r^3-r^5\Lambda/3)+\sum_{k=1}^{n-1}(-1)^k\frac{[(2n-3)(2n-5)...(2n-2k-1)]r^3-[(2n-5)(2n-7)...(2n-2k-3)]r^5\Lambda/3}{2^k[(n-1)(n-2)...(n-k)]\rho_0^{2k}(\rho_0^2-r^2)^{-k}}\right]\right\}\Bigg|_{r_0}^{r}+$$

$$+2n(r^2-\rho_0^2)^n(-1)^{n+2}\left[\rho_0^{1-2n}\frac{(2n-3)(2n-5)...3}{2^n n!}\left(\frac{r}{\rho_0}-\frac{1}{2}\ln\left|\frac{r+\rho_0}{r-\rho_0}\right|\right)-\rho_0^{3-2n}\Lambda\frac{(2n-5)(2n-7)...3}{3\cdot 2^n n!}\left(\frac{r^3}{3\rho_0^3}+\frac{r}{\rho_0}-\frac{1}{2}\ln\left|\frac{r+\rho_0}{r-\rho_0}\right|\right)\right]\Bigg|_{r_0}^{r}$$

Thus, the quantum transition of cooling down matter into its new quasiequilibrium state is accompanied by the increment by one step of the indicators of degree of all components of polynomial equation of gravitational field:

$$n+1=[(3+S_n/R_{Tn})-\widetilde{\beta}_{pVn}(1-S_n/R_{Tn})]/[(1+S_n/R_{Tn})-\widetilde{\beta}_{pVn}(1-S_n/R_{Tn})]=2/[(1+S_{(n+1)}/R_{T(n+1)})-\widetilde{\beta}_{pV(n+1)}(1-S_{(n+1)}/R_{T(n+1)})],$$

where: $\widetilde{\beta}_{pV(n+1)}=\widetilde{\beta}_{pVn}(1-S_n/R_{Tn})/[2-\widetilde{\beta}_{pVn}(1-S_n/R_{Tn})]$, $S_{(n+1)}/R_{T(n+1)}=(\widetilde{\beta}_{pVn}-1)(1-S_n/R_{Tn})/[(3+S_n/R_{Tn})-\widetilde{\beta}_{pVn}(1-S_n/R_{Tn})]$.

So, in the cooled down to the limit state of matter ($S=0$) the parameter $\widetilde{\beta}_{pV}=pV/U_0$ takes its minimal possible value that is equal to one. Therefore: $pV\geq U_0$. In case of small values of indicators of polynomial function the cooling down cluster of gas dumps its energy in big portions. However these portions of energy of radiation become less and less with each new taken geavithermodynamic state. Thus, the gas cluster can cool down for infinitely long time. The precise solution of this differential equation when $1/a_0=0$ is as follows:

$$\frac{1}{u}=\frac{b^{\widetilde{\beta}_H/2\widetilde{\beta}_{pV}}}{\kappa\widetilde{\mu}_c c^2 a}=-\frac{r^2-r_0^2}{2n}+\frac{1+3\widetilde{\beta}_{pV}}{2\widetilde{\beta}_{pV}}\int_{r_0}^{r}\frac{r_g-2\Lambda r^3/3}{r'_g}dr=\frac{r_{eff}^2-r_{0eff}^2}{2n},$$

where: $r_{eff}<r$ and $r_{0eff}<r_0$ are effective values of radial distance that are significantly smaller than real values on the big distances from the center of gravity. It is possible that exactly this causes the effect of stronger gravitational field (than it is according to Newton's theory) and, therefore, causes the false necessity of dark non-baryonic matter in the Universe.

Conclusion

The quantum equation of gravitational field have been found, the solutions of which set the spatial distribution of gravitational radius of matter in its every new gravithermodynamic state with the polynomial function with the next more high degree. The indicator of the degree of this function of continuously cooling down matter can successively take only integer and semi-integer values. That is why the process of cooling down of the whole GTD-bonded matter is the quantum process that is caused by its spontaneous transition to the polynomial function with more high value of degree and, therefore, to the next quantum collective state.

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