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RELATIVISTIC THERMODYNAMICS WITH LORENTZ-INVARIANT EXTENSIVE VOLUME

Introduction

Inherent in the classical physics ordinary notions about the absolute simultaneity of events and about uniqueness of concept of time[1], as well as of determination of spatial volume which moving body fills in, essentially hinder us from the formation of the most perfect relativistic generalization of thermodynamics. Purely logically-mathematical approach to the solution of the problems, which don't permit to get the full interconsistency of thermodynamics special (SR) and general (GR) relativities, can't guarantee the positive result of theoretical research in principle. To attain the aim we need to make philosophical remaking sense of many of our physical notions, which are only conceived as finally established and unshakable. This work is the attempt of construction of relativistic thermodynamics, based on the rejection of some dogmata, inherent not only in classical physics but also in well-known relativistic generalizations of thermodynamics.

It is considered that SR itself doesn't lead to the unique concept of the temperature, attributed to moving body [1,2]. Therefore a few relativistic generalizations of thermodynamics with lorentz-invariant pressure are known. First of all there are Planck-Hasenöhrl relativistic thermodynamics [3–5] and Ott relativistic thermodynamics [6], which are equally based on the lorentz-invariance of entropy and pressure but use essentially different transformations of temperature and heat [2,7]. According to Planck and Hasenöhrl, moving body is "colder" than motionless [1]. On the contrary, according to Ott transformations, moving body is "hotter" than motionless. Thermodynamics with lorentz-invariant relativistic temperature [1,8] is attractive for the fact that in this thermodynamics temperatures of phase transitions are the intrinsic properties of substances as they are in classical thermodynamics. However the equations of this thermodynamics don't lead to such conversion of the energy of radiation that corresponds to the relativistic Doppler shift of the frequency of radiation.

In most relativistic generalizations of thermodynamics the linear momentum of moving body is used as an additional extensive parameter. However, in contrast to mechanics, in relativistic thermodynamics this linear momentum is considered to be proportional to the enthalpy H of matter [1,9,10] but not to the internal energy U, which is equivalent to the eigenvalue of the mass of matter. Therefore this linear momentum forms the four-vector with hamiltonian of enthalpy but not with the hamiltonian of energy of the matter [1,10]. And as D'Alembert pseudoforce of inertia[2] is the value derived from the linear momentum, in fact it is proposed to use the enthalpy of matter instead of its mass as the measure of inertness.

In some of the relativistic generalizations of thermodynamics, along with invariant pressure, noninvariant forms of relativistic pressure are proposed. In such noninvariant forms of relativistic pressure the presence of mechanical linear momentum [9] and heat exchange [11,12] for moving body is taking into account.

1. Nonextensivity of relativistic molar volume

Two "equal in rights" relativistic values of molar volume of the matter, which is moving at the velocity $v=dx/dt$ in the external frame of references of spatial coordinates and time (FR), are possible in principle. These values are: Lagrangian volume $\mathrm{v}_R = \mathrm{v}/\Gamma$, which in common three-dimensional space is filled in by the world points of moving matter (these world points correspond to the same moment of

[1] There are two different times in GR: standard time and coordinate time. Analogously in SR: relativistic standard (path-like) time, which determines "individual ages" of the objects of matter, and coordinate time, which determines kinematics and dynamics of the motion of these objects in the space.

[2] Total thermodynamic value of mechanical pseudoforce of inertia, which takes into account in Planck thermodynamics expenditure of energy not only on the increasing of the linear momentum of matter, but also on the relativistic shrinkage of geometrical volume (pseudocontraction) of this matter, is proportional to $(2H–U)$ near the point of zero value of the velocity. As example, for monoatomic ideal gas this value more than two times (7/3 times) exceeds the value of the pseudoforce of inertia, which is determined in classical mechanics. As this fact is not confirmed by any experiments, relativistic generalizations of thermodynamics, which are based on these physical notions, can't be considered as physical reality.

time t of the external FR); and Hamiltonian volume[3] $v_R^*=v\Gamma$, which in this space is filled in by the world points of moving matter (these world points correspond to the same moment of relativistic standard time t^* (which is identical with the proper time t_0 of matter) of this matter and, consequently, to the same collective space-time state of this matter [13,14]). Here: v – value of molar volume of the matter in comoving FR (eigenvalue of molar volume); $\Gamma=(1-v^2/c^2)^{-1/2}$ – relativistic dilation of physical processes and time in the moving; c – constant (eigenvalue) of the velocity of light.

In classical thermodynamics eigenvalue of molar volume v(S, p) is strictly extensive parameter and its change causes the change of enthalpy, and consequently the change of rates of physical processes, not directly, but via change of entropy S and eigenvalue of pressure p. Relativistic values of molar volume v_R and v_R^* are not strictly extensive parameters and, therefore, can't be equivalent to nonrelativistic value (eigenvalue) of molar volume v. Relativistic shrinkage of the length and, cinsequently, of the molar volume v_R is aimed at guaranteeing of isotropy of the frequency of electromagnetic interaction of the molecules, atoms and elementary particles of matter and, thus, it is aimed at the guaranteeing of isotropy of the rate of physical processes in moving matter [13,14]. Therefore, change of this shrinkage has the direct influence on the relativistic value of enthalpy and on the rates of physical processes and rates of proiper time of moving matter. Moreover, relativistic decrease of molar volume of the matter is not accompanied by overcoming of any forces of resistance to it[4] and, consequently, it takes place inertially and in strong coordination with the change of the velocity of matter. Therefore, relativistic shrinkage of body length and of molar volume of the matter of this body are considered in SR as purely kinematic effect, which is not accompanied by expenditure of energy on the execution of work on relativistic "selfcontraction" of matter. And if the energy was expended not only on the increasing of the value of linear momentum but also additionally on its "selfcontraction", for the purely dynamic consideration of the matter (this consideration doesn't take into account these additional expenditure of energy) law of conservation of energy wouldn't be fulfilled in the mechanics.

All this is the sufficiently strong reason for the use of only strictly extensive value of molar volume $(v_R^*\cdot v_R)^{1/2}=v$, which is equal to the eigenvalue of molar volume in classical thermodynamics[5], in relativistic equations of equilibrium state of matter. Lorentz-invariance of energy density[6] ε and of equivalent to it mass density $\mu=\varepsilon/c^2$ is the consequence of lorentz-invariance of the strictly extensive value of molar volume.

This gives the possibility to make the most simple definition of lagrangian (according to [9], internal energy of matter U_R, transported by the body): $L=-U_R=-\varepsilon v_R=-\varepsilon v/\Gamma$ and hamiltonian (total

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[3] Hamiltonian volume is the mapping of volume, which occupies one mole of matter on three-dimensional space-like hypersurface (which is the section of Minkowski space when $t^*=const$), onto three-dimensional space Its value is deyermined by the velocity of propagation of the front of proper time of moving matter $u=c^2/v$ in the external FR The fact that in relativistic generalizations of thermodynamics lagrangian volume is used instead of hamiltonian is the consequence of the classical notions about the absolute simultaneity of events and is connected with the neglect of nonsimultaneity of the same collective space-time state of moving matter in different points of the space of FR, not comoving with matter.

[4] Rigid body, which can't be deformed by as big as desired force in statics, can be relativistic-deformed by appreciably smaller force that accelerates its motion. And, consequently, relativistic selfcontraction of molecular volume of the matter is not the result of the influence of any forces on it, but the adaptation of molecules, atoms and elementary particles of this matter to changed conditions of their interaction. This also can be considered as the reaction of self-sustaining wave packets (soliton-like spiral-wave formations), wich correspond to the elementary particles of matter, on the increasing of the velocity of their propagation.

[5] This, however, doesn't mean that relativistic values of molar volume v_R and v_R^* are unclaimed in relativisyic thermodynamics. They are used at the transition from spatial densities of thermodynamic characteristics to integral values of these characteristics. And as the self-organization of equilibrium thermodynamic states takes place quasisynchronously in proper time t_0 of moving matter, in GR, which examines processes and results of self-organization of spatially inhomogeneous equilibrium states of matter, integration is executed over the v_R^* and not over the lagrangian relativistic volume v_R, which is often examined in relativistic thermodynamics. Thus integral value of total energy U_R^*, which corresponds to the same collective space-time state of matter, is being determined.

[6] Not the density of amount of matter, but densities of relativistic energy and equivalent to it relativistic mass are lorentz-invariant. After all, different by the value lagrangian and gamiltonian molar volumes contain the same amount of the molecules of matter.

energy of matter[7], which is conserved in the process of inertial motion of the body): $U_R^*=\varepsilon v_R^*=\varepsilon v\Gamma$. Lagrangian is the contravariant component and hamiltonian – covariant component of four-momentum of matter. They correspond to conjugate spaces. Lagrangian corresponds to contravariant pseudoeuclidean Minkowski space and characterizes decelerated rate ($dt=\Gamma(ds)_{x0}=\Gamma dt^*=\Gamma(dt_0)_{x0}$) of physical processes in matter and, consequently, in points of comoving with matter space[8] ($dx_0=0$). Hamiltonian corresponds to covariant euclidean four-dimensional space, in which velocity of motion of matter and rates of physical processes in matter are determined by external observer not by his own clock, but by comoving with matter clock. By the clock of external observer hamiltonian characterizes the accelerated ($(dt)'=(ds)_x=dt^*/\Gamma$) rates of physical processes directly in the space, in which observer rests ($dx=0$). After all, based on the fact that elementary particles of matter are nonmechanicaly self-excited microstates of undragable by motion physical vacuum (Dirac "sea") [13,14,16], all physical processes and the motion itself (motion is considered in quantum physics as the quantum process of successive change of space-time states of microobjects of matter and, consequently, of microstates of physical vacuum) can be considered as the processes that take place also directly in not comoving with matter space. Here $(ds)_{x0}$ – is the increment of the interval between world points of the events that is equal (because of $dx_0=0$) to the increment of proper time of matter and, consequently, to the increment $dt^*=dt/\Gamma$ of relativistic standard time, which is being read in external FR by comoving with matter clock. $(ds)_x$ – is the increment of the interval between world points of the events that is equal (because of $dx=0$) to the increment of time t of FR, in which motion of this matter is observed. It is essential to notice that effective values of densities of examined here relativistic energies: $\varepsilon_R=U_R/v=\varepsilon/\Gamma$ and $\varepsilon_R^*=U_R^*/v=\varepsilon\Gamma$, are, of course, not lorentz-invariant.

2. Noninvariance of the pressure

According to Noether theorem [17] the law of conversation of energy is the consequence of the presence of symmetry for time and the fulfillment of this law is possible only for homogeneity of time. This homogeneity of time is in the invariance of physical laws relatively to the change of the start of time reading and this homogeneity is guaranteed by the use of uniform scale, by which rates of physical processes in matter in its identical thermodynamic states are equal in any moment of time, for its measuring. According to this, mutual complementarity of energy and time, which is declared by Bohr principle of complementarity and becomes apparent in the presence of Heisenberg uncertainty relation of these physical characteristics, takes place.

In the system of units of measurment of physical magnitude, which is based on dimensionless Planck constant h and, thus, reflects the presence of mutual complementarity of energy and time, the dimension of pressure [$\mathrm{sec}^{-1}\mathrm{m}^{-3}$] denotes the following. In not comoving with matter FR of exterior observer the value of pressure, as well as the value of energy (dimension [sec^{-1}]), must depend on the rate of time in this FR. According to GR in rigid FR only proportional synchronization of clocks, time rates by which in different points of space with different values of gravitational potential are not equal, is possible. In accordance with this, value of pressure in any point j of such physically inhomogeneous space is determined by rates of physical processes not only in this point but also in the point i, from which observation takes place [16]: ${}_j^i p={}_j^i v_c \cdot p=(v_{cj}/v_{ci})p$. Here v_{cj} and v_{ci} – set by gravitational field improper gravibaric (coordinate-like [10]) values of the velocity of light in the points j and i of matter, which is in spatially inhomogeneous equilibrium thermodynamic state [15,18]. Therefore if we want the pressure in matter to remain intensive parameter[9], its relativistic value must be unequal in different IFR, which are moving relatively to this matter at unequal velocities, and its transformations must be the same as for all other intensive parameters. And, consequently, relativistic value of pressure in

[7] The considering of covariant time component of four-momentum, instead of contravariant, as the total energy allows to avoid in GR the problem of nonconservation of total energy and allows to not consider the difference between contravariant and covariant general relativistic values of energy as the energy, collectivized in gravitational field [15].

[8] It is essential to differ observed in external FR comoving with matter space from motionless in comoving with matter FR intrinsic space of this matter.

[9] In Planck thermodynamics relativistic molar volume and pressure in fact are as if they exchange the roles. Regarding relativistic transformations molar volume begin to "behave" as intensive parameter, while pressure – as extensive parameter.

moving matter, which is determined in Minkowski space and in fact attributed to comoving with this matter space, must be the same times smaller than its eigenvalue as observed by external observer rate of physical processes in this matter and, consequently, in comoving with it space: $p_R=p/\Gamma$. To the contrary, relativistic value of pressure, which is determined in covariant world space and attributed to resting three-dimensional observer space, must be the same times greater than its eigenvalue: $p_R^*=p\Gamma$. After all, in not comoving with matter FR of external observer rates of physical processes in the points of this space are the several times greater than in comoving with matter FR. And these processes take place the same times smaller for external observer than for observer, resting in comoving with matter FR. The conception of lorentz-invariance of pressure is connected with the substitution of the extensive value of molar volume v for nonstrictly extensive lagrangian value v_R of this volume in relativistic differential equations of equilibrium state of matter.

3. Main differential equations of relativistic thermodynamics in contravariant representation

Equations of relativistic thermodynamics are usually determined in coordinates of contravariant world space (Minkowski space) and, therefore, covariant relativistic value of energy (hamiltonian) is being expressed not via covariant relativistic value of enthalpy $H_R^*=H\Gamma$, but via contravariant relativistic values of enthalpy $H_R=H/\Gamma$ and thermodynamic parameters attributed to moving matter:

$$U_R^* = U_R + P^* v = (H_R - \mathrm{v}p_R) + P^* v, \qquad U_R^* dt - P^* dx = U_R\, dt = (H_R - \mathrm{v}p_R)dt = Udt^*.$$

Here, taking into account all stated above, total relativistic energy $U_R^*=U\Gamma$ (haniltonian) and covariant value of mechanical linear momentum of matter[10] $P^*=-(\partial U_R/\partial v)_{S,\mathrm{v}}=w/v=U\Gamma v/c^2=U_R^* v/c^2$, analogously to its internal energy U, don't depend on pressure directly, and therefore, are strictly extensive characteristics. At that, increment of hamiltonian, as well as increment of internal energy, is determined by the increments of only strictly extensive parameters:

$$dU_R^* = (TdS - pd\mathrm{v})\Gamma + U_R^* d\ln\Gamma = T_R \cdot dS - p_R\, d(\Gamma \mathrm{v}_R) + vdP^* = T_R \cdot dS - p_R d\mathrm{v} + vdP^*$$

and no work on relativistic "selfcontraction" of matter is executed. Here: $w=vP^*=U\Gamma v^2/c^2$ – external energy (energy of tranfer [9]) of matter, $T_R=T/\Gamma$ – contravariant relativistic value of temperature (Planck temperature [1,2,5]) of moving body.

In contrast to Planck linear momentum, mechanical linear momentum (which doesn't depend on pressure directly) $P^*= U_R^* v/c^2$ together with hamiltonian U_R^* form four-momentum. And any limitations of the dependence of coordinate (gravibaric [15]) value of the velocity of light v_c in matter on thermodynamic parameters of this matter[11] are not required. D'Alembert pseudoforce of inertia, the same way as in mechanics, doesn't directly depend on pressure in matter and may be represented in gauge-invariant form:

$$\mathbf{F}_{in} = -(\partial \mathbf{P}^*/\partial t)_{S,\mathrm{v}} = -(\partial U_{Rg}^*/\partial \Gamma)_{S,\mathrm{v},v_c} \cdot d\Gamma/d\mathbf{x} = -(U_{Rg}^*)d\ln\Gamma/d\mathbf{x},$$

as well as gravitational pseudoforce[12]:

$$\mathbf{F}_g = -(\partial U_{Rg}^*/\partial v_c)_{S,\mathrm{v},\Gamma} \cdot \mathbf{grad} v_c = -U_{Rg}^* \cdot \mathbf{grad}(\ln v_c),$$

where: $U_{Rg}^*=U_R^* \cdot v_c/c=U\Gamma v_c/c$ – total energy[13] (general relativistic covariant value of hamiltonian) of matter, which is being conserved (Γv_c=const) in the process of body free fall. The possibility to gauge-transform v_c, as well as Γ , not only allows to additively sum up pseudoforces of gravity and inertia, but also guarantees mutual independence of their hamiltonian strengths.

[10] These expressions for energy and linear momentum has been derived by Brotas [11,12]. However, he cosiders them not as total relativistic values of energy and linear momentum, but only as components of total values of these characteristics determined in proper time of matter.

[11] Because of this, we ought to consider examined in [15] six-momentum, which is based on Planck relativistic generalization of thermodynamics, only as artificial construction, which is caused by initially wrong presuppositions and physical notions.

[12] Here gradient of natural logarythm of improper value of light velocity is being determined taking into account the curvature of intrinsic space of matter, or in other words, not via photometrical but via metrical increments of coordinates.

[13] According to this, total energy is equivalent not to contravariant general relativistic mass $m_{Rg}=m\Gamma c/v_c$, but to covariant $m_{Rg}^*=m\Gamma v_c/c$ mass, which determines not time but relative spatial inertness of matter, where m – eigenvalue of mass. The fact that gravitational pseudoforce, as well as D'Alembert inertia pseudoforce, is proportional to the total energy makes the problem of equivalence of gravitational and inert masses unactual.

The increment of contravariant relativistic value of enthalpy[14]:

$$dH_R=(TdS+\mathrm{v}dp)/\mathit{\Gamma}-H_R d\ln\mathit{\Gamma}=T_R\cdot dS+\mathrm{v}_R\cdot\mathit{\Gamma}dp_R-P^*dv=T_R\cdot dS+\mathrm{v}dp_R-P^*dv,$$

as well as increment of ordinary enthalpy H, is determined by increments of only intensive parameters (except, of course, the increment of entropy).

4. Main differential equations of relativistic thermodynamics in covariant representation

In FR, in which the motion of matter is observed, differental equations of relativistic thermodynamics may be also determined in coordinates of covariant world space:

$$U_R dt^*+Pdx=U_R^* dt^*=(H_R^*-\mathrm{v}p_R^*)dt^*=Udt.$$

At that, contravariant relativistic value of energy (lagrangian) $U_R=U/\mathit{\Gamma}=U\mathit{\Gamma}^*$ may also be expressed not via contravariant relativistic value of enthalpy $H_R=H/\mathit{\Gamma}=H\mathit{\Gamma}^*$, but via covariant relativistic values of enthalpy $H_R^*=H\mathit{\Gamma}=H/\mathit{\Gamma}^*$ and thermodynamic parameters of matter, attributed to the areas of space, filled in with this moving matter:

$$U_R=UP/P^*=U_R^*-Pv^*=H_R^*-\mathrm{v}p_R^*-Pv^*,$$

where: $\mathit{\Gamma}^*=(1+v^{*2}/c^2)^{-1/2}=1/\mathit{\Gamma}$; $P=(\partial U_R^*/\partial v^*)_{S,\mathrm{v}}=U\mathit{\Gamma}^*v^*/c^2=U(\partial U_R^*/\partial P^*)_{S,\mathrm{v}}=Uv/c^2$ – contravariant value of mechanical linear momentum[15]; $v^*=(\partial x/\partial s)_{x0}=dx/dt^*=\mathit{\Gamma}v$ – spatial component of four-velocity, which in fact is the velocity of motion of matter determined in observer FR not by its intrinsic clock, but by relativistic standard clock, comoving with this matter. According to this:

$$P^*=Uv^*/c^2,\qquad Pv^*=P^*v,\qquad U_R^{*2}-c^2P^{*2}=U_R^2+c^2P^2=U_R\,U_R^*=U^2,$$

and:

$$U_R^*-U_R=P^*P/U=-P(\partial U_R/\partial P)_{S,\mathrm{v}}=P^*(\partial U_R^*/\partial P^*)_{S,\mathrm{v}}$$

Main differential equations of relativistic thermodynamics in their contravariant representation have the following form:

$$dH_R^*=(TdS+\mathrm{v}dp)/\mathit{\Gamma}^*-H_R^*d\ln\mathit{\Gamma}^*=T_R^*\cdot dS+\mathrm{v}dp_R^*+Pdv^*,$$

$$dU_R=(TdS-pd\mathrm{v})\mathit{\Gamma}^*+U_R d\ln\mathit{\Gamma}^*=T_R^*\cdot dS-p_R^*d\mathrm{v}-v^*dP.$$

Here: $T_R^*=T/\mathit{\Gamma}^*=T\mathit{\Gamma}$ – Ott relativistic temperature [1,2,6,7], which corresponds to the accelerated rates of physical processes in the points of space where matter is in the state of motion. Thus, areas of space filled in with moving matter become "hotter"[16] [1] than coincidal with them areas of intrinsic space of matter. Any pair of correlated photons, which in comoving with matter FR have the same energy U_{c0} and propagate in it strictly in opposite directions, in not comoving with matter FR have total energy $\mathit{\Gamma}$ times greater than in comoving with matter FR: $U_c+U_c'=2U_{c0}\mathit{\Gamma}=2U_{c0}/\mathit{\Gamma}^*$, where:
$U_c=U_{c0}\mathit{\Gamma}^*[1-(v/c)\cos\varphi]^{-1}=U_{c0}\mathit{\Gamma}[1+(v/c)\cos\varphi_0]$; $U_c'=U_{c0}\mathit{\Gamma}^*[1-(v/c)\cos\varphi']^{-1}=U_{c0}\mathit{\Gamma}[1+(v/c)\cos(\varphi_0+\pi)]$; φ_0 and $(\varphi_0+\pi)$ – angles in comoving with matter FR between directions of propagation of photons and direction of matter motion; φ and φ' – corresponding to them angles in not comoving with matter FR, in which directions of propagation of correlated photons are not parallel in general case.

This, as well as numerous works [2,6,7,10], which confirmed the validity of consideration of Ott relativistic temperature $T_R^*=T\mathit{\Gamma}$ side by side with Planck temperature $T_R=T/\mathit{\Gamma}$, denotes appropriateness of consideration of covariant relativistic generalization of thermodynamics side by side with contravariant one.

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[14] Bazarov [1] adduces the similar expression (which doesn't contain the increment of velocity) for differential of lorentz-invariant enthalpy, and thus, he in fact examines thermodynamic state of moving matter not in external FR, in which motion is observed, but in comoving with this matter FR.

[15] In contrast to examined in GR [10] coordinate contravariant value of linear momentum, relativistic standard contravariant value of linear momentum is being determined not by coordinate clock of external FR, but by relativistic standard clock, comoving with matter and, therefore, observed in external FR as moving. The expression for determination of covariant relativistic value of linear momentum strictly coincides with corresponding expression of classical physics.

[16] To put it more precisely: undragable by motion physical vacuum becomes "hotter", if the motion is observed in its fundamental FR. After all, elementary particles of matter are just nonmechanicaly excited microstates of physical vacuum – space-time modulations of its main characteristics in the form of self-organized soliton-like spiral-wave formations [13,14,16].

Conclusions

Examined here relativistic generalization of thermodynamics with strictly extensive molar volume and lorentz-invariant entropy and energy of density is devoid of many disadvantages of relativistic generalizations with lorentz-invarisnt pressure and allows us to newly interpret perception of course of physical processes in moving body from not comoving with it FR. Possibility of two complementary representations of differential equations of relativistic thermodynamics (contravariant and covariant) solves the problem of the presence of two alternative relativistic temperatures – Planck temperature and Ott temperature.

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